Strongly Nonlinear Effect in Unstable Wakes

B. J. A. Zielinska,[2,1] S. Goujon-Durand,[1,3] J. Dušek,[4] and J. E. Wesfreid[1]
[1]*Ecole Supérieure de Physique et Chimie Industrielles de Paris (ESPCI), PMMH-URA CNRS No. 857, 10, rue Vauquelin, 75231 Paris Cedex 05, France*
[2]*Gas Engineering Montrouge/Schlumberger Ind., B.P. 620-13, 92542 Montrouge Cedex, France*
[3]*Laboratoire d'Electronique, Université Paris XII, 61 Av du Géneral de Gaulle, Créteil Cedex, France*
[4]*Université Louis Pasteur, IMF-URA CNRS No. 854, 2, rue Boussingault, 67000 Strasbourg, France*
(Received 19 December 1996; revised manuscript received 28 May 1997)

We perform experiments and numerical simulations on the mean streamwise velocity field of an unstable cylinder wake. We show that the mean velocity exhibits two weak secondary minima as the consequence of nonlinear interactions, resulting in a strong mean flow correction of the unstable mode. This correction, dominating the basic flow, governs the decrease in the length of the recirculation region ΔL_r in the supercritical regime. This explains the early classical observations of M. Nishioka and H. Sato [J. Fluid Mech. **89**, 49 (1978)]. [S0031-9007(97)04455-4]

PACS numbers: 47.27.Vf, 47.20.Ky

For many decades the classic example of flow around a circular cylinder presented a kaleidoscope of challenging phenomena, many of which still remain unsolved, in particular, the behavior governing the size of the eddies behind the body. In the current work we present a series of experimental and numerical data that show a surprising spatial behavior of the mean flow in unstable wakes. We study the mean (time averaged) longitudinal component of the velocity field in the vortex shedding regime. We compare this stationary field with the basic (steady) flow around the obstacle, and we focus our analysis on the spatial properties of the mean flow corrections produced by the hydrodynamic instability.

In the case of a circular cylinder a zone of negative values of the streamwise velocity (the recirculation zone or separated flow) appears as soon as the Reynolds number Re reaches a value of about 5 [1]. The prediction of separated circulation in slow viscous flows around cylinders is a long-standing problem in applied mathematics, especially in the theory of matched asymptotic expansions. Recently Keller and Ward [2] gave a first estimation of the critical Reynolds number, where the twin eddies appear to be $\mathrm{Re} = 2.4$, a lower value that is observed experimentally and numerically. As the Reynolds number increases in the steady laminar regime, the streamwise profiles of the streamwise (x axis) velocity component $U(x, 0)$ (on the $y = 0$ axis) present an increasingly longer and deeper recirculation [1,3,4]. The (negative) minimum reached in this zone is the only local minimum of the profile. Past this minimum, the profile increases monotonically to reach, asymptotically, the inflow velocity value. The size of the recirculation zone is given by $U(x, 0) = 0$. At the Reynolds number Re_c of about 46, the Bénard–von Kármán instability sets in. The flow is no longer steady and symmetric with respect to the plane defined by the cylinder axis and the inflow velocity direction. However, the mean flow conserves this symmetry and allows one to generalize the length of the recirculation zone L_r on the basis of the longitudinal profile of the mean (time averaged) streamwise velocity. At supercritical Reynolds numbers the recirculation length starts to decrease, as was observed by Nishioka and Sato [5]. Until now, no explanation was given for this strong decrease in $L_r = L_r(\mathrm{Re})$. In the present paper we show that this decrease is the result of a strong mean flow correction of the unstable mode.

Furthermore, we show that at sufficiently high supercritical Reynolds numbers the mean streamwise velocity profile no longer grows monotonically downstream of the recirculation zone but exhibits two consecutive minima, the second of which was reported by Williamson and Prasad [6]. Their experimental investigation of the far wake mean velocity deficit provides evidence of a minimum far downstream (about 50 cylinder diameters) of the recirculation zone. Here we present evidence of the first minimum located at about 11 cylinder diameters, which to our knowledge was not previously reported.

To analyze more deeply the phenomena connected to the decrease of L_r for $\mathrm{Re} > \mathrm{Re}_c$, we have carried out experiments and numerical simulations in confined and unconfined wake flow. The obtained experimental and numerical results are explained on the basis of the recently proposed nonlinear theory of the Bénard–von Kármán instability [7] by analyzing separately the mean value, the basic flow, and the nonlinear mean value correction.

Numerical simulations were carried out using a spectral element solver NEKTON. We simulated an unconfined and a confined 2D wake representing the experimental configuration. For the unconfined case the same spectral element mesh and high resolution as that used in Refs. [7] and [8] were applied, which allowed us to bring the numerical errors below 1% (see [9]). The characteristics of the cylinder wake obtained in [7,8] compare extremely well with known experimental data, namely, those of Williamson [10]. The confined 2D configuration is a rectangular domain that is 16.6 cylinder diameters large and 100 diameters long. The confined flow simulation can thus be considered to be of the same numerical accuracy as that of the unconfined flow. The interest of this numerical experiment is to observe the same physical trends as in real experiments.

In the unconfined domain, where $\mathrm{Re}_c = 46$, we have simulated the oscillating and basic flow for $\epsilon = (\mathrm{Re} - \mathrm{Re}_c)/\mathrm{Re}_c = 2.26$. The stationary basic flow was obtained by forcing the symmetry with respect to the x axis by imposing a zero transverse velocity at $y = 0$. In Fig. 1(a) we plotted the basic flow and the mean (time averaged) value of the streamwise velocity on the symmetry axis. The observed mean velocity is negative immediately downstream from the obstacle. Further downstream, the mean velocity becomes positive and passes through two minima denoted M1 and M2 before assuming the growing trend at the exit of the domain. The basic flow grows monotonically downstream of the recirculation region. The difference between both profiles, called the *mean flow correction*, is also plotted in this figure. This correction is quite large. Close to the recirculation region the maximum of the mean flow correction is larger than the mean oscillating flow velocity. The rapid increase of the basic flow velocity causes the secondary minimum M1 in the mean flow profile at about 11 diameters downstream of the cylinder. Furthermore, we recover, at about 50 diameters downstream, the secondary minimum M2 reported experimentally by Cimbala [11] and Williamson and Prasad [6], resulting from the minimum of the mean value correction. As a result, the profile has a wavy character with as much as three local minima: that lying in the recirculation zone very close to the cylinder, and two secondary minima at 11 and 50 cylinder diameters downstream.

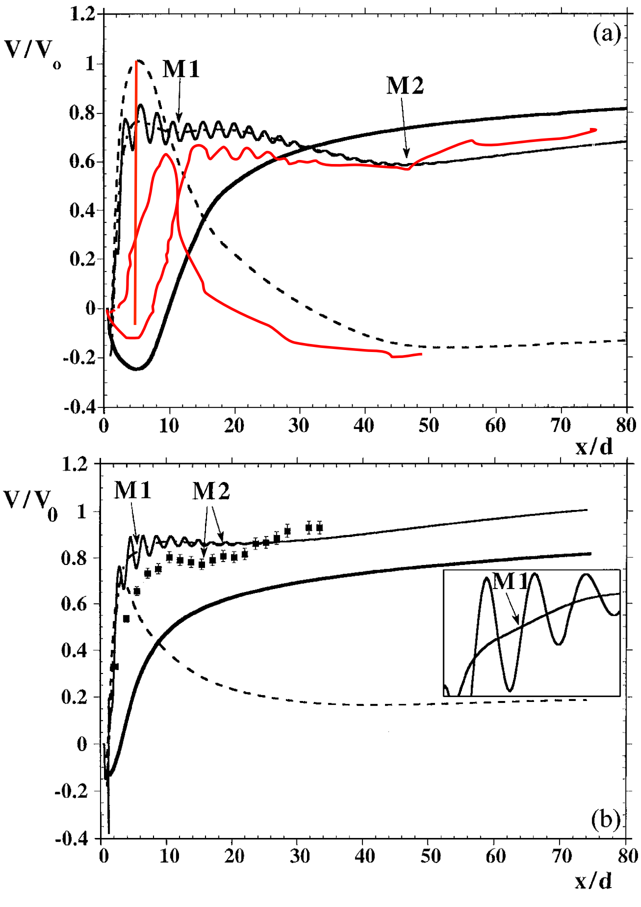

FIG. 1. Longitudinal profile of the streamwise mean velocity (dash-dotted line), basic flow (thick solid line), instantaneous values (solid line), and mean value correction (dashed line); (a) for unconfined cylinder and $\epsilon = (\mathrm{Re} - \mathrm{Re}_c)/\mathrm{Re}_c = 2.26$ (V_0 is the inflow velocity) and (b) for confined cylinder and $\epsilon = (\mathrm{Re} - \mathrm{Re}_c)/\mathrm{Re}_c = 0.73$ (V_0 is the average of the inflow velocity). M1 and M2 denote the secondary minima in the mean velocity profile. The inset in (b) enlarges the vicinity of the point M1.

The position of the last minimum agrees well with that found in [11] and [6]. The velocity deficit found by simulation (0.4 of the inflow velocity) agrees rather well with that in [11]. We found that this secondary minimum appears as early as $\epsilon = 0.3$, about 40 diameters downstream of the cylinder, and its distance from the cylinder increases slightly with increasing Reynolds number. The velocity deficit grows with the Reynolds number: It is of 0.2 of the inflow velocity at about 40 diameters for $\epsilon = 0.3$, of 0.3 at about 45 diameters for $\epsilon = 1.17$, and 0.4 at about 50 diameters for $\epsilon = 2.26$.

It should be noted that, 32 diameters and further downstream, the basic flow streamwise velocity becomes larger than the oscillating flow velocity. This means that on the symmetry axis the instability accelerates the flow in the first $30d$ and slows it down further downstream as a consequence of a nonlinear saturation mechanism [12].

In Fig. 1(a) we have also plotted the instantaneous streamwise velocity profile showing that, downstream of the minimum at 50 diameters, the oscillations of the global mode of the second harmonic [13,14], the fundamental mode being zero on the symmetry axis, are imperceptible.

A 2D computational domain with experimental geometry and velocity profile, which was flat in the center and had stick boundary conditions at the side walls, was also considered. The critical Reynolds number $\mathrm{Re}_c = 62$ was calculated on the basis of the flat velocity value in the center at the inflow. Subsequently, this velocity was corrected for the acceleration of the flow due to confinement (6% increase in our case, see [15]). For $\epsilon = 0.73$ the mean velocity profile obtained in numerical simulations is similar to the unconfined case [Fig. 1(b)]. The first secondary minimum after the recirculation zone degenerates to a local minimum of the slope and is situated at about $6d$ downstream [see inset in Fig. 1(b)]. This can be explained by a significantly smaller mean flow correction maximum. The second minimum M2 occurs at about $18d$ and, similarly, to the $\epsilon = 2.26$ unconfined case it also corresponds to the end of the global mode. The fact that this minimum occurs much closer to the obstacle is due to the proximity of the lateral walls, which prevent the wake from spreading.

We have performed experiments in a large blockage ratio ($d/l_y = 1/16.6$) low velocity water tunnel of square section with $l_z = l_y = 5$ cm (the experimental setup is described in [14,16]) and a cylindrical obstacle attached directly to the side walls. Here $d = 3$ mm is the diameter of the bluff body, l_y is the vertical, and l_z is the spanwise extension of the tunnel. We measured the streamwise velocity with a laser Doppler anemometer. We found that the critical Reynolds number is equal to 59.8 ± 7, estimated with the average velocity at the inflow. The probe scans the flow along the central plane ($y = 0$) from $x = d/2$ (the downstream face of the bluff body) until $x = 34d$, in steps of $1.6d$. We have measured the mean velocity profile along the central $y = 0$ plane. Although the experimental flow is characterized by 3D effects at the considered aspect ratio, a very good qualitative agreement with the confined 2D simulation was found [see Fig. 1(b)] showing that the described phenomena are not restricted 2D flows. The second secondary minimum M2 is clearly visible at $15d$, while the minimum M1 is only suggested by the data.

We turn now to a more detailed study of the change of the recirculation length due to the development of the instability. For this purpose we have plotted, in Fig. 2(a), L_r for the basic and mean unstable flow as a function of the Reynolds number. Indeed while L_r grows linearly for the basic flow, it starts decreasing as soon as the flow becomes unstable (here for Re = 46). For the basic flow we have found $L_r/d = 0.0670(\pm 0.0008)\mathrm{Re} - 0.405(\pm 0.035)$ giving $\mathrm{Re} = 6.0 \pm 0.5$ for the onset of the recirculation, in agreement with earlier results [1]. The difference in L_r increases with the Reynolds number, suggesting that the mean flow correction becomes more important. This difference ΔL_r depends on the distance from the instability threshold, and the best fit of the data gives a power law $\Delta L_r/d \sim \epsilon^{0.84}$ which is close to a linear dependence on the Reynolds number [Fig. 2(b)]. A similar scaling was also observed recently in [17]. Therefore it is the decrease in the recirculation length ΔL_r that characterizes the Bénard–von Kármán instability and not the absolute length of the recirculation, for which no scaling law as a function of ϵ can be found, as was shown in [13]. This decrease in size of the eddies acts as a global order parameter.

Let us take a stream function formulation of the 2D Navier-Stokes equations as a starting point in order to reduce the flow characterization to just one scalar variable.

In [7] it was shown that, if the unsteady Navier-Stokes solution describing the instability is decomposed into the sum $\psi = \psi_0 + \varphi$ of the steady but unstable flow ψ_0 and of the oscillating perturbation φ, the latter can be expanded into the Fourier series,

$$\varphi(t) = \sum_{n=-\infty}^{+\infty} c_n(t) e^{in\omega t}, \tag{1}$$

with ω expressing the angular frequency of the wake. The Navier-Stokes equation for the perturbation written in a compact form as

$$\frac{\partial \nabla^2 \varphi}{\partial t} + \mathcal{L}[\psi_0]\varphi + \mathcal{B}(\varphi, \varphi) = 0, \tag{2}$$

with $\mathcal{B}$ and $\mathcal{L}[\psi_0]$ standing, respectively, for the bilinear operator defining the nonlinearity, and the Navier-Stokes operator can then be replaced by the following system of coupled nonlinear equations for the Fourier modes c_n:

$$\frac{\partial \nabla^2 c_n}{\partial t} + (\mathcal{L}[\psi_0] + in\omega\nabla^2)c_n + \sum_{k=-\infty}^{+\infty} \mathcal{B}(c_k, c_{n-k}) = 0. \tag{3}$$

Equation (3) can easily be solved numerically [18] and analyzed theoretically [7] due to the fact that the modes $c_n = \overline{c_{-n}}$ have been shown to be rapidly decreasing as functions of n.

This simple approach allows one to explain the phenomena described above. The mean value of the wake flow field is expressed as the sum of the (unperturbed) basic flow and the mean value of the nonlinear perturbation φ:

$$\langle \psi \rangle = \psi_0 + c_0. \tag{4}$$

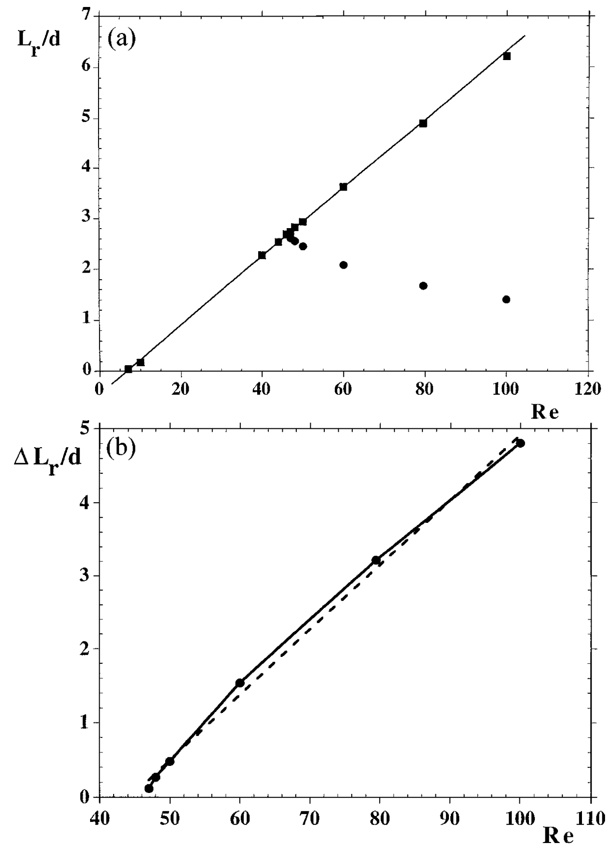

FIG. 2. Reynolds number dependence of (a) the length of the recirculation zone L_r/d (unconfined circular cylinder). Circles: mean value; squares: basic flow. (b) The decrease of the recirculation length $\Delta L_r/d$. Solid line: best power law fit; dashed line: best linear fit.

In [7] it has been shown that, at least in the neighborhood of the threshold, the dominant mode is the fundamental $n = 1$, the maxima of the higher harmonics being roughly proportional to the nth power of the ratio of the maximum of the amplitude of the fundamental and the inflow velocity. Extensive numerical analysis [8,18] confirmed the validity of this statement.

As far as the mean value correction is concerned, it is proportional to the square of the magnitude of the fundamental close to the threshold; however, it soon becomes extremely significant, as shown in Figs. 1(a) and 1(b). Already quite close to the threshold, this correction is non-negligible (see [8]). At twice the threshold and above, it dominates the wake. Contrary to the mean oscillating flow, the basic flow continues to have the expected shape far above the threshold, and its evolution with increasing Reynolds number follows the expected trend with a growing recirculation zone [see Fig. 2(a)—squares] and slowing downstream growth of the profile. The nonlinear correction c_0 is primarily generated by the fundamental, and has thus a similar simple shape. In contrast, their sum [dash-dotted lines in Figs. 1(a) and 1(b)] can be quite complex. The trend of the mean streamwise velocity to increase or decrease is the result of the balance of the downstream slopes of the basic flow and the nonlinear mean velocity correction, and depends strongly on the configuration and the Reynolds number. The nonlinear mean flow correction reaches values comparable to the inflow velocity [see Fig. 1(a)], which explains why it can lead to qualitatively different spatial structures of the mean and basic flows, and the Reynolds number dependence of the recirculation zone plotted in Fig. 2.

In conclusion, we have shown the existence of two secondary minima in the mean longitudinal velocity profile downstream of the bluff body, in the vortex shedding regime in both numerical simulations and experimental data. We demonstrate that the deformation of the basic flow is generated by the presence of the zeroth Fourier component of the velocity perturbation playing the role of mean flow correction. Our results imply that the nonlinear effects in unstable wakes appear to be extremely significant at already moderate Reynolds numbers, relegating linear and weakly nonlinear considerations only to the relatively restrained domain of problems treating the very onset of the instability. The presented analysis shows that the aspect of the mean oscillating flow is the result of nonlinear coupling between the basic flow and the fundamental unstable mode.

The physical explanation of the supercritical decrease in L_r is the existence of the strong mean flow correction concentrated within the recirculation zone of the basic flow that can be interpreted as a source of positive velocity pumping inside the region of negative velocity. This pumping shortens the recirculation zone and brings the saddle point of the velocity field closer to the shedding body. Consequently, the linear growth of the recirculation length L_r in the subcritical region is stopped and L_r starts to decrease, explaining an old and classical result of fluid mechanics [5]. We show that this decrease in the recirculation length ΔL_r is a characteristic order parameter of the Bénard–von Kármán instability and scales with ϵ. Indeed, for the stationary flow ($\mathrm{Re} < \mathrm{Re}_c$) $L_r \sim V_0$ (where V_0 is the basic flow), while for ($\mathrm{Re} > \mathrm{Re}_c$)—the supercritical case—$\langle V \rangle = V_0 - \delta V$, where δV is the nonlinear correction. This nonlinear correction is of the second order in the parameter of the instability (the amplitude of the oscillations $A \sim \epsilon^{1/2}$), so $\delta V \sim A^2 \sim \epsilon$. Consequently, $L_r \sim (V_0 - \epsilon)$. From this simple consideration we deduce that $\Delta L_r \sim \epsilon$, which is close to the result obtained in numerical simulations.

A fully nonlinear theoretical approach to the instability analysis leads to considering the nonlinear correction of the mean flow due to the instability onset. A combination of numerical simulations and velocity measurements presented here allowed us to quantify its role, showed its extreme significance, and provided an explanation for several well-known but puzzling features of wakes.

[1] M. Coutanceau and R. Bouard, J. Fluid Mech. **79**, 231 (1977), and references within.
[2] J.B. Keller and M.J. Ward, J. Eng. Math. **30**, 253 (1996).
[3] S. Acrivos, L.G. Leal, D.D. Snowden, and F. Pan, J. Fluid Mech. **34**, 25 (1968).
[4] B. Fornberg, J. Comput. Phys. **61**, 297 (1985).
[5] M. Nishioka and H. Sato, J. Fluid Mech. **89**, 49 (1978).
[6] C.H.K. Williamson and A. Prasad, J. Fluid Mech. **256**, 269 (1993).
[7] J. Dušek, P. Le Gal, and Ph. Fraunié, J. Fluid Mech. **264**, 59 (1994).
[8] J. Dušek, Euro. J. Mech. B, Fluids **15**, 619 (1996).
[9] J. Dušek and Ph. Fraunié, Proceedings of the 8th International Conference on Numerical Methods in Laminar and Turbulent Flow, Swansea, 1993 (to be published).
[10] C.H.K. Williamson, J. Fluid Mech. **206**, 579 (1989).
[11] J.M. Cimbala, Ph.D. thesis, California Institute of Technology, 1984 (unpublished).
[12] A. Maurel, V. Pagneux, and J.E. Wesfreid, Europhys. Lett. **32**, 217 (1995).
[13] B.J.A. Zielinska and J.E. Wesfreid, Phys. Fluids **7**, 344 (1995).
[14] J.E. Wesfreid, S. Goujon-Durand, and B.J.A. Zielinska, J. Phys. II (France) **6**, 1343 (1996).
[15] C. Mathis, Ph.D. thesis, Université d'Aix-en-Provence, France, 1983 (unpublished).
[16] S. Goujon-Durand, J.E. Wesfreid, and P. Jenffer, Phys. Rev. E **50**, 308 (1994).
[17] L. Socolescu, Ph.D. thesis, Université du Havre, France, 1996 (unpublished).
[18] G. Carte, J. Dušek, and Ph. Fraunié, J. Comput. Phys. **120**, 171 (1995).